Eur. Phys. J. B **9**, 567–568 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Erratum

Non-equilibrium critical behavior of O(n)-symmetric systems

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Eur. Phys. J. B **7**, 309-330 (1999)

In Section 4.3 of reference [1], we have unfortunately absorbed a factor $c_0^{d_\parallel/2}$ in the definitions of the geometric factors $A(d_\parallel, d_\perp)$ and $B(d_\parallel, d_\perp)$, instead of correctly including this factor in the definition of appropriate renormalized couplings for the non-equilibrium model J. However, the coupling c_0 in general renormalizes non-trivially, which leads to a modified expression for the RG beta functions. Equations (4.18) and (4.50) of reference [1] should thus be replaced with

$$\widetilde{A}(d_\parallel, d_\perp) = \frac{\Gamma(3-d/2-d_\parallel/2)\Gamma(d/2)}{2^{d-1}\pi^{d/2}\Gamma(d_\perp/2)},$$
$$\widetilde{B}(d_\parallel, d_\perp) = \frac{\Gamma(4-d/2-d_\parallel/2)\Gamma(d/2)}{2^{d}\pi^{d/2}\Gamma(d_\perp/2)}. \tag{1}$$

The relation between the renormalized (dimensionless) coupling constants $\widetilde{u}$ and $\widetilde{g}$ and $\widetilde{u}_0$ and $\widetilde{g}_0$ is now given by

$$\widetilde{u} = Z_{\widetilde{u}}\widetilde{u}_0\widetilde{A}(d_\parallel, d_\perp)\mu^{-\epsilon},$$
$$\widetilde{g} = Z_{\widetilde{g}}^{1/2}\widetilde{g}_0\widetilde{B}(d_\parallel, d_\perp)^{1/2}\mu^{-\varepsilon/2}. \tag{2}$$

Next, we define the correct bare effective coupling constants $\widetilde{v}_0$ [2] and $\widetilde{f}_0$ according to

$$\widetilde{v}_0 = \frac{\widetilde{u}_0}{c_0^{d_\parallel/2}}, \quad \widetilde{f}_0 = \frac{\widetilde{g}_0^2}{2d_\perp\lambda_0^2 c_0^{d_\parallel/2}}, \tag{3}$$

with their renormalized counterparts given by

$$\widetilde{v} = \frac{\widetilde{u}}{c^{d_\parallel/2}}, \quad \widetilde{f} = \frac{\widetilde{g}^2}{2d_\perp\lambda^2 c^{d_\parallel/2}}. \tag{4}$$

With these definitions, the expressions (4.52) to (4.58) in reference [1] have the same functional dependence on $\widetilde{v}_0$, $\widetilde{f}_0$, $\widetilde{A}(d_\parallel, d_\perp)$, and $\widetilde{B}(d_\parallel, d_\perp)$ as they had on $\widetilde{u}_0$, $\widetilde{f}_0$, $A(d_\parallel, d_\perp)$, and $B(d_\parallel, d_\perp)$. In particular, in order to obtain the correct Wilson zeta functions, we merely need to substitute $\widetilde{u}$ by $\widetilde{v}$ and $\widetilde{f}$ as given in (4) into equations (4.60) to (4.65) in reference [1]. The correct beta functions to one-loop order then read

$$\beta_{\widetilde{v}} = \widetilde{v}\left(\zeta_{\widetilde{u}} - \frac{d_\parallel}{2}\zeta_c\right)$$
$$= \widetilde{v}\left(-\epsilon + \frac{11}{6}\widetilde{v} - \frac{d_\parallel(104-38d_\parallel+3d_\parallel^2)}{12(4-d_\parallel)}\widetilde{f}\right), \tag{5}$$

and

$$\beta_{\widetilde{f}} = 2\widetilde{f}\left(\zeta_{\widetilde{g}} - \zeta_\lambda - \frac{d_\parallel}{4}\zeta_c\right)$$
$$= \widetilde{f}\left(-\varepsilon - \frac{3d_\parallel^3 - 44d_\parallel^2 + 176d_\parallel - 96}{12(4-d_\parallel)}\widetilde{f}\right), \tag{6}$$

where we have used the fact that $\zeta_c = -\zeta_\lambda$, as given by equation (4.63) of reference [1]. To one-loop order, the finite fixed point

$$\widetilde{f}^* = \frac{12(4-d_\parallel)}{96-176d_\parallel+44d_\parallel^2-3d_\parallel^3}\varepsilon + O(\varepsilon^2, \epsilon^2) \tag{7}$$

is positive only in the interval $0 \le d_\parallel \le 0.644838$. For $d_\parallel = 1$, the RG flow takes the coupling constant to infinity. Therefore, the conclusions drawn in reference [1] remain valid. However, the (somewhat unphysical) expressions for the critical exponents in the expansion with respect to ϵ, ε and $d_\parallel$ need to be corrected. To first order in $d_\parallel\varepsilon$, we now have

$$\widetilde{f}^* = \frac{\varepsilon}{2}\left(1 + \frac{19}{12}d_\parallel\right), \quad \widetilde{v}^* = \frac{6}{11}\epsilon + \frac{13}{22}d_\parallel\varepsilon, \tag{8}$$

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leading to the critical exponents

$$\eta = -\frac{3}{8}d_\parallel\varepsilon, \quad \eta_S = -\frac{1}{2}d_\parallel\varepsilon, \tag{9}$$

$$\nu^{-1} = 2 - \frac{5}{11}\epsilon - \frac{65}{132}d_\parallel\varepsilon, \tag{10}$$

$$z = 4 - \frac{\varepsilon}{2} - \frac{7}{24}d_\parallel\varepsilon, \tag{11}$$

$$\Delta = 1 - \frac{\varepsilon}{4} - \frac{7}{48}d_\parallel\varepsilon. \tag{12}$$

We have also repeated the computation of the RG flow function β_f at *fixed* dimensions d and $d_\parallel$, which yields that in the physical dimension $d = 3$, the non-trivial zero of $\beta_{\widetilde{f}}$ is given by $\widetilde{f}^* = -24/49$ for $d_\parallel = 1$, and $\widetilde{f}^* = -6/52$ for $d_\parallel = 2$ (while $\widetilde{f}^* = 0$ for $d = 2$ and $d = 4$). Thus, for any positive initial value of $\widetilde{f}$, the RG flow runs away to ∞ ("strong coupling") as in the ε expansion. We note that in the case of the two-temperature non-equilibrium model B, there is no renormalization of c_0 to one-loop order, and therefore the results given in reference [1] carry through.

References

1. U.C. Täuber, J.E. Santos, Z. Rácz, Eur. Phys. J. B **7**, 309 (1999).
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